P 13
4 12
5 12
6 9
7 1

4, 5, 6, 7, P

Name .....

Number.....

HSC Mathematics

Assessment Task 1 - 2016

November 2015 Time Allowed - 60 minutes +5 mins reading

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice	/4
Question 5	/10
Question 6	/ 9
Question 7	/12
Question 8	/12
Total	/47

Fill in the correct answer on the answer sheet - Questions 1 - 4 are worth 1 mark each

1. A parabola has a focus of $(-a,\ 0)$ and a directrix at $x = a$. Its equation is:

A $y^2 = 4ax$

B $y^2 = -4ax$

C $x^2 = 4ay$

D $x^2 = -4ay$

2. The derivative of $\frac{2}{x^2}$ is

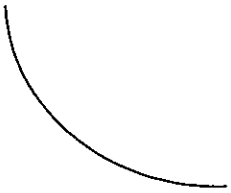
A $-4x^{-3}$

B $\frac{1}{x}$

C $\frac{-1}{x^3}$

D $\frac{-1}{2x^3}$

3. For the curve,



A
$f'(x) > 0$ and $f''(x) < 0$

B
$f'(x) > 0$ and $f''(x) > 0$

C
$f'(x) < 0$ and $f''(x) < 0$

D
$f'(x) < 0$ and $f''(x) > 0$

4. The gradient of the tangent to $y = 2x^3 - 3x - 5$ at $(-1, 2)$ is

A -2

B 3

C $\frac{1}{2}$

D $-\frac{1}{3}$

Question 5 (10 Marks) (Begin a new sheet of paper) **Marks**

a) Differentiate i) $y = 4\pi + 4x^3 - 7x - 2$ **2**

ii) $y = x\sqrt{x}$ **2**

H 5

b) Find the derivative in simplest factorised form of

i) $y = (2 - 3x)^7$ **2**

ii) $y = \dfrac{2x - 1}{(x+1)^2}$ **4**

H 5

Question 6 (9 Marks) (Begin a new sheet of paper)

For the curve $y = 5x^3 - 3x^5$

i) Show that the x intercepts are $x = 0$ and $x = \pm 1.3$ to 1 decimal place. **1**

ii) Find the coordinates of the stationary points. **2**

iii) Determine the nature of each stationary point. **3**

iv Sketch the curve. (half page approximately) **3**

H 6

Question 7 12 Marks (Begin a new sheet of paper)

a) Prove that the locus of a point which is equidistant from the point $(-1, 2)$ and the line $y = -2$ is a parabola and find its equation. **3**

b) For the parabola $(y+1)^2 = 4(x-2)$:

i) Find the coordinates of the vertex **1**

ii) Find the coordinates of the focus **1**

iii) Find the equation of the directrix **1**

iv) Sketch the parabola showing all these features. **2**

c) Find the equation of the locus of a point which is equidistant from $3x + 4y = 12$ and the y axis. **4**

Question 8 (12 Marks) (Begin a new sheet of paper)

a) A rectangular prism has edges x metres, $2x$ metres and y metres. The sum of all 12 edges is 36 metres.

i) Show that $y = 9 - 3x$ **1**

ii) Show that its volume is given by $V = 18x^2 - 6x^3$. **2**

iii) Hence find the maximum volume. **3**

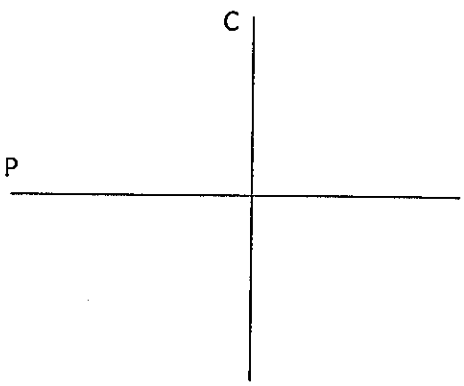
b) Two people set out from towns on roads which meet at right angles and walk towards the intersection. Pat is 25 km from the intersection and walks at 4 km/h. Chris is 20 km from the intersection and walks at 3 km/h.

i) Show that the distance D km apart after t hours is **2**

$$D = \sqrt{25t^2 - 320t + 1025}$$

ii) Find the time in hours and minutes when Pat and Chris are closest. **3**

iii) Hence find their minimum distance apart. **1**

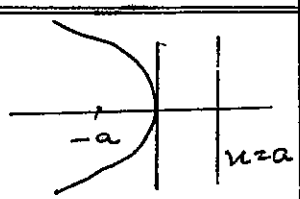


END OF TEST

Solutions to 2016 Mathematics Task 1

1. B

$y^2 = -4ax$



2 $y = 2x^{-2}$

$y' = -4x^{-3}$ A

3 $f' < 0$, $f'' > 0$ D

4 $y' = 6x^2 - 3$

$x = -1$, $m = 6 - 3 = 3$ B

Question 5

a) i) $y' = 12x^2 - 7$

ii) $y = x^{3/2}$

$y' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

b) i) $y' = 7(2-3x)^6 \cdot -3$

$= -21(2-3x)^6$

ii) $y' = \frac{(x+1)^2 \cdot 2 - (2x-1) \cdot 2(x+1)}{(x+1)^4}$

$= \frac{2(x+1)\{x+1-2x+1\}}{(x+1)^4}$

$= \frac{2(2-x)}{(x+1)^3}$

Question 6

i) $y = 5x^3 - 3x^5$

$y = 0 \quad x^3(5-3x^2) = 0$

$x = 0$ or $x^2 = \frac{5}{3}$

$x = 0$ or $x = \pm\sqrt{\frac{5}{3}} \doteq \pm 1.3$

ii) $y' = 15x^2 - 15x^4$

$= 15x^2(1-x^2)$

$= 15x^2(1-x)(1+x)$

Stationary points when

$x = 0$, $x = 1$, $x = -1$

$y = 0$, $y = 2$, $y = -2$

iii) Nature of (0, 0)

x	0^-	0	0^+
y'	+	0	+

(0, 0) is a horizontal point of inflexion.

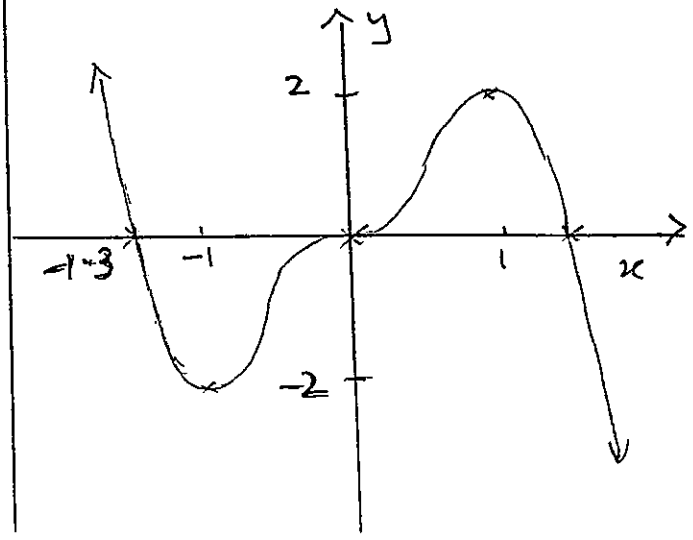
$y'' = 30x - 60x^3$

when $x = 1$, $y'' = -30$ ∩

Negative ∴ (1, 2) is a maximum.

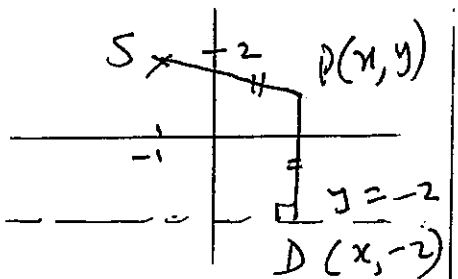
when $x = -1$ $y'' = 30$ ∪

positive ∴ (-1, -2) is a minimum.



Question 7

a)



$PS = PD$

$PS^2 = PD^2$

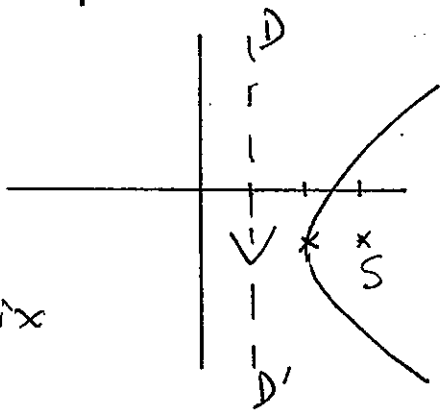
$(x+1)^2 + (y-2)^2 = (x-x)^2 + (y+2)^2$

$(x+1)^2 + y^2 - 4y + 4 = y^2 + 4y + 4$

$(x+1)^2 = 8y$

which is a parabola.

b)

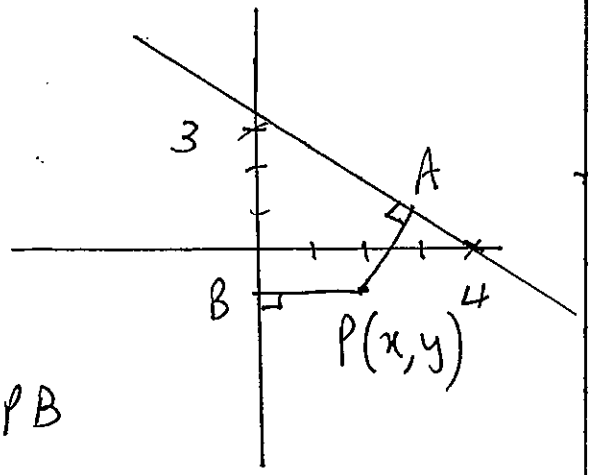


i) $V(2,-1)$

ii) $S(3,-1)$

iii) Directrix $x = 1$

c)



$PA = PB$

$$\left| \frac{3x+4y-12}{\sqrt{3^2+4^2}} \right| = |x|$$

$3x+4y-12 = 5x$

$0 = 2x - 4y + 12$

$0 = x - 2y + 6$

OR

$3x+4y-12 = -5x$

$8x+4y-12 = 0$

$2x+y-3 = 0$

$\therefore$ Locus is a pair of perpendicular lines

$\begin{cases} x-2y+6=0 \\ 2x+y-3=0 \end{cases}$ &

bisecting the angles of intersection.

Question 8

a) $4(x+2x+y) = 36$

$3x+y = 9$

$y = 9-3x$

ii) $V = x \cdot 2x \cdot y$

$= 2x^2(9-3x)$

$V = 18x^2 - 6x^3$

iii) For maximum volume $\frac{dV}{dx} = 0$

$36x - 18x^2 = 0$

$18x(2-x) = 0$

$x = 0$ or 2

$\frac{d^2V}{dx^2} = 36 - 36x$

When $x = 2$

$\frac{d^2V}{dx^2} = 36 - 72$

Negative $\therefore$ Maximum.

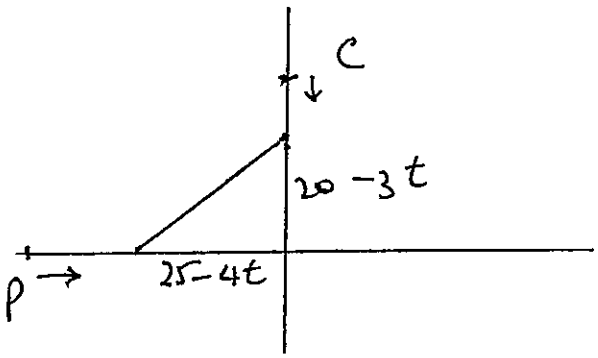
$V = 18 \times 4 - 6 \times 8$

$V = 24\ m^3$

8 b)

$S = \frac{D}{T}$

$D = ST$



In t hours

Pat travels 4t Km

∴ she is now $(25-4t)$ Km from intersection

In t hours

Chris travels 3t Km

She is now $(20-3t)$ Km from intersection

Distance apart is

$D^2 = (25-4t)^2 + (20-3t)^2$

$= 625 - 200t + 16t^2 + 400 - 120t + 9t^2$

$D^2 = 25t^2 - 320t + 1025$

$D = \sqrt{25t^2 - 320t + 1025}$

ii) For minimum distance, $\frac{dD^2}{dt}$ is 0

$\frac{d(D^2)}{dt} = 50t - 320$

$t = \frac{320}{50} = 6.4$ hrs.

$= 6$ hrs 24 mins.

$\frac{d^2D^2}{dt^2} = 50t$ which is positive when t is positive. ∴ D^2 is minimum

ie D is minimum.

iii) $D = \sqrt{25 \times 6.4^2 - 320 \times 6.4 + 1025}$

$= \sqrt{1} = 1$

Minimum distance apart is 1 Km.